Orientational pinning of quantum Hall striped phase

E. E. Takhtamirov[*] and V. A. Volkov[†]

Institute of Radioengineering and Electronics of RAS,
11 Mokhovaya, Moscow 103907, Russia

In ultra-clean 2D electron systems on (001) GaAs/AlGaAs upon filling high Landau levels, it was recently observed a new class of collective states, which can be related to the spontaneous formation of a charge density wave ("striped phase"). We address to the following unsolved problem: what is the reason for stripe pinning along the crystallographic direction [110]? It is shown that in a single heterojunction (001) A_3B_5 the effective mass of 2D electrons is anisotropic. This natural anisotropy is due to the reduced (C_{2v}) symmetry of the heterojunction and, even being weak (0.1%), can govern the stripe direction. A magnetic field parallel to the interface induces "magnetic" anisotropy of the effective mass. The competition of these two types of anisotropy provides quantitative description of the experiment.

I. INTRODUCTION

It had been assumed [1] in 1979 that a uniform 2D electron system in strong magnetic fields corresponding to the filling of the lowest Landau level, $N = 0$ (filling factor $\nu < 1$), can be unstable against the formation of a 1D charge density wave with a period of the order of magnetic length. This instability is due to the exchange interaction resulting in the effective attraction between electrons. The analysis was carried in the Hartree-Fock approximation, which overestimates the exchange interaction and ignores electron-electron correlations. After the discovery of the fractional quantum Hall effect, it became clear that, for $\nu < 1$, it is the correlation interaction that leads to the formation of a uniform state of the Laughlin liquid type. Nevertheless, the role of correlations diminishes with filling a large number of Landau levels and, in principle, one can expect the indicated instability to appear. It was predicted in 1996 [2] that a 1D charge density wave may appear near the half-filling of Landau levels, beginning with $N = 2$ [3]. Such a striped phase with a period of order of the Larmor diameter should be energetically more favorable than the Laughlin liquid and the Wigner crystal [2,3,4].

How should this phase be manifested in the transport measurements if it is really formed and pinned for some

reason? Similar problem was considered, probably, for the first time in a series of old papers [5,6,7], where the anisotropic conductivity was calculated [5] and the high-frequency [6] and heating [7] effects were studied. In the presence of a periodic 1D potential $U(x)$ induced by the charge density wave, each Landau level transforms into a narrow 1D band. At the edges of this band, the density of states has a power divergence, which is cut off upon including weak scattering. As a result, the density of states $S(E_f)$ at the Fermi level has the shape of a two-teeth fork: a minimum in the center of the band and two peaks at the band edges, the lower the density n_{im} of scatterers, the higher the peaks. Incomplete filling of this band results in the formation of stripes differing in the ν value [of the type $\nu/(\nu-1)/\nu/...$] and aligned with the y-axis. The transverse (σ_{xx}) and longitudinal (σ_{yy}) conductivities obey different mechanisms and qualitatively differently depend on ν: the σ_{yy} conductivity is high, has the band character, and is inversely proportional to n_{im} and $S^2(E_f)$, while σ_{xx} is low, has the hopping character, and is proportional to n_{im} and $S^2(E_f)$, with the product $\sigma_{xx}\sigma_{yy}$ being independent of scattering. These results were in fact confirmed and generalized in [8,9]. What did the experiment actually reveal?

In 1999, the conductivity of an electron system with ultrahigh mobility in the (001) GaAs/AlGaAs structures was studied at very low temperatures near half-integer $\nu \geq 9/2$, and a new state was revealed [10,11,12,13] which was assumed to be just the one associated with the formation of the striped phase predicted in [2]. This assumption was primarily based on the observation of a giant resistance anisotropy in this system. The ratio of resistances along the crystallographic directions [1$\bar{1}$0] and [110] reaches the values of $R_{xx}/R_{yy} \sim$ 5-3500, depending on the sample geometry, where [110] is the "easy" conductivity direction. Moreover, the behavior of all conductivity tensor components qualitatively agrees with the theory [5,8,9]. The predicted behavior of the $\sigma_{xx}\sigma_{yy}$ product near the half filling of the upper Landau level numerically agrees with the experiment [14]. It was also shown in [12,13] that the magnetic field $B_\parallel \sim 1\,\text{T}$ parallel to the interface can change the direction of easy conductivity. The authors of [12] concluded that, at high enough $B_\parallel$, the direction of easy conductivity is perpendicular to the $B_\parallel$ direction. Similar result was obtained in [13] for $\mathbf{B}_\parallel \| [110]$ near all half-integer $\nu \geq 9/2$ and for $\mathbf{B}_\parallel \| [1\bar{1}0]$ near $\nu = 11/2$ and $\nu = 15/2$. The theoretical analysis [15,16] of the influence of $\mathbf{B}_\parallel$ carried out in the Hartree-Fock approximation in the model of parabolic quantum well partially explained the results. All this is strong ev-

[*]e-mail:gibbon@royal.net
[†]e-mail:VoVA@mail.cplire.ru

idence for the formation of a striped phase. However, a mechanism responsible for the orientation of the charge density stripes along a certain preferred direction in a macroscopic sample (orientational pinning) at $B_\parallel = 0$ remains to be clarified. This is one of the fundamental unsolved problems.

In this work, the Kroemer's assumption [17] that the reduced (C_{2v}) symmetry of the potential in a heterostructure based on semiconductors without inversion center can cause an appearance of a preferred direction for the conductivity is justified. Symmetry reduction means that the cubic axis normal to the interface is transformed from the fourfold mirror-rotational axis (S_4) to the twofold axis (C_2). We demonstrate below that, owing to the asymmetry of the potential of an atomically sharp heterojunction, the effective mass (EM) of 2D electrons is anisotropic (natural anisotropy). At the same time, the presence of $\mathbf{B}_\parallel$ also gives rise to the EM anisotropy (magnetic anisotropy) [18]. Therefore, the results of many-particle numerical calculations of the $\mathbf{B}_\parallel$ effect on the orientation of striped phase [15,16] is natural to treat (to the lowest order in $B_\parallel^2$) as a manifestation of the magnetic anisotropy of EM. Thus, the many-particle problem of orientational pinning of the striped phase reduces to a one-particle problem of determining the EM anisotropy. We derive below analytical expressions for both types of EM anisotropy (natural and magnetic) and demonstrate that they can compete with each other. At a certain magnitude and direction of $\mathbf{B}_\parallel$, these two types of anisotropy exactly cancel, leading to the disappearance of resistance anisotropy, in agreement with the experiment.

II. NATURAL ANISOTROPY

Before proceeding to the many-particle problem, it is necessary to obtain one-particle Hamiltonian for the conduction band in a (001) A_3B_5 heterostructure. As was shown in [19], the correctly constructed multiband set of equations for the envelope functions retains information on the heterostructure symmetry (C_{2v}), which is lower than the symmetry T_d of the constituent materials. This symmetry reduction is described, in particular, by certain short-range potentials localized at the heterointerface. Mixing of heavy and light holes at the center of the 2D Brillouin zone is one of the consequences of symmetry reduction [19,20]. This mixing explains giant optical anisotropy (with the same principal axes [110] and [1$\bar{1}$0]) that was discovered in [21] for the quantum wells based on semiconductors with different cations and anions. Evidently, low symmetry should also manifest itself in the equation for the envelope functions in the conduction band. Nevertheless, a single-band equation obtained in [19] carries no information on the C_{2v} symmetry because the corresponding small contributions were neglected. We must now take them into account. Since the terms with the symmetry higher than C_{2v} are of no

interest here, the effective Hamiltonian can include only the operators of kinetic and potential energies used in the standard EM approximation, as well as the anisotropic contribution of C_{2v} symmetry, which will be obtained below.

A single-band Hamiltonian of the C_{2v} symmetry can be obtained by the method of invariants. Leading aside spin-orbit interaction, one can conclude that the C_{2v} symmetry should manifest itself in the kinetic energy operator. Let us direct the 2D quasi-momentum components along the cubic axes: $p_x \| [100]$ and $p_y \| [010]$ (z axis is along [001]). Then the part of kinetic energy operator quadratic in the generalized 2D momentum (P_x, P_y) should be

$$T = \frac{P_x^2 + P_y^2}{2m^*} + \frac{1}{2}\mathcal{A}\left(P_x P_y + P_y P_x\right). \tag{1}$$

Here, m^* is the EM of conduction band, and the quantity $\mathcal{A}$ (which may depend on z) accounts for the natural anisotropy of EM in the plane of 2D electron gas. Let us obtain explicit expression for $\mathcal{A}$ by using a multiband matrix Hamiltonian [19] acting on the column of envelope functions. In the $\mathbf{k}$ representation, it takes the form

$$H_{nn'}^{(eff)} = H_{nn'}^{(kp)} + \frac{1}{2\pi}D_{0nn'}, \tag{2}$$

where n and n' are the band indices. The first term in Eq. (2) includes the contributions from the smooth potentials and $\mathbf{kp}$ interaction and has the standard form. The second term in Eq. (2) is a contribution from the atomically sharp heterointerface potential taken to a first order in the small parameter $\bar{k}a$, where $1/\bar{k}$ is the characteristic length of changing the envelope functions and a is the lattice constant. One can pass to the single-band variant of envelope-function method by applying the perturbation theory, with the $\mathbf{kp}$ and $\mathbf{D}_0$ operators as perturbation. The second order in the $\mathbf{kp}$ interaction gives the first (standard) term in Eq. (1). The third order (second order in $\mathbf{kp}$ and first in $\mathbf{D}_0$) provides the second term of Eq. (1), with $\mathcal{A}(z) = \alpha\delta(z)$ and

$$\alpha = \sum_{n,n'}' \frac{2\langle c \mid p_x \mid n\rangle D_{0nn'}\langle n' \mid p_y \mid c\rangle}{m_0^2\left(\epsilon_c - \epsilon_n\right)\left(\epsilon_c - \epsilon_{n'}\right)}$$
$$+ \sum_{n,n'}' \frac{4D_{0cn}\langle n \mid p_x \mid n'\rangle\langle n' \mid p_y \mid c\rangle}{m_0^2\left(\epsilon_c - \epsilon_n\right)\left(\epsilon_c - \epsilon_{n'}\right)}. \tag{3}$$

Here $\delta(z)$ is the Dirac δ-function, $z = 0$ defines the heterointerface position, $\langle n \mid p_i \mid n'\rangle$ is the ith component of the interband momentum matrix element, c is the index of conduction band, m_0 is the mass of free electron, and ϵ_n is the energy of the nth band edge in one of the structure materials. In the simplest model, the key parameters of the theory ($D_{0nn'}$) have the form

$$D_{0nn'} = \sum_{j=\pm 1,\pm 2,\ldots} \frac{\langle n \mid \delta U \sin(4\pi jz/a) \mid n'\rangle}{4\pi j/a}$$

$$\times \int_{-\infty}^{+\infty} \frac{dG(z)}{dz} \cos\left(\frac{4\pi}{a} jz\right) dz. \tag{4}$$

The functions $G(z)$ and $\delta U(\mathbf{r})$ are so defined that the crystal potential of the heterostructure has the form $U(\mathbf{r}) = U_1(\mathbf{r}) + G(z)\delta U(\mathbf{r})$, where U_1 and $U_2 = U_1 + \delta U$ are the crystal potentials of the structure materials. Note that the D_{0XY} parameter accounts for the mixing of the heavy and light holes at the center of the 2D Brillouin zone, with X and Y being the indices of the Bloch functions corresponding to the edge of the Γ_{15} valence band and transforming as x and y under symmetry operations of the T_d group [19,20].

III. INCLUSION OF THE MAGNETIC ANISOTROPY

Reducing the tensor of reciprocal effective mass to the principal axes, so that $x\|[1\bar{1}0]$ and $y\|[110]$ in the new co-ordinates, and introducing magnetic field $\mathbf{B}$ in the vector-potential gauge $\mathbf{A} = (B_y z, -B_x z + B_z x, 0)$, one obtains for the orbital part of the 3D Hamiltonian of conduction band

$$H_{3D} = V(z) + \frac{p_z^2}{2m^*} + \frac{1}{2}\left(\frac{1}{m^*} - \alpha\delta(z)\right)\left(p_x + \frac{e}{c}B_y z\right)^2$$
$$+ \frac{1}{2}\left(\frac{1}{m^*} + \alpha\delta(z)\right)\left(p_y - \frac{e}{c}B_x z + \frac{e}{c}B_z x\right)^2. \tag{5}$$

Here, $V(z)$ is the effective potential of the conduction band edge, e is the elementary charge, and c is the light speed (we hope, it will be no confusion with the conduction band index). For the finite thickness of the 2D layer, the magnetic field component parallel to the heterointerface can be treated perturbatively [18]. To second order in $B_\|$, this procedure brings about a diamagnetic shift of the dimensional-quantization subbands and an increase (for the lowest subband) in EM in the direction perpendicular to $\mathbf{B}_\|$. The natural EM anisotropy also can be treated perturbatively. For simplicity, we assume that $\mathbf{B}_\|$ is parallel to either $[1\bar{1}0]$ or $[110]$, so that $B_x B_y = 0$. Collecting all terms second-order in $B_\|$ and first-order in α, one obtains the following expression for the 2D Hamiltonian of the lowest subband:

$$H_{2D}^1 = E_1 + \frac{e^2}{2m^*c^2}\left(B_x^2 + B_y^2\right)\left(\langle z^2 \rangle_{11} - \langle z \rangle_{11}^2\right)$$

$$+ \frac{1}{2m^*}\left[1 - \frac{\Delta_{nat}}{2} - \frac{B_y^2}{B_\|^2}\Delta_B\right]\left(p_x + \frac{e}{c}B_y \langle z \rangle_{11}\right)^2 \tag{6}$$

$$+ \frac{1}{2m^*}\left[1 + \frac{\Delta_{nat}}{2} - \frac{B_x^2}{B_\|^2}\Delta_B\right]\left(p_y + \frac{e}{c}B_z x - \frac{e}{c}B_x \langle z \rangle_{11}\right)^2.$$

The parameters of the natural EM anisotropy and the EM anisotropy induced by the magnetic field are

$$\Delta_{nat} = 2m^*\alpha \langle \delta(z) \rangle_{11}, \quad \Delta_B = \frac{2e^2 B_\|^2}{m^*c^2}\sum_m{}'\frac{|\langle z \rangle_{1m}|^2}{E_m - E_1}, \tag{7}$$

where E_m is the energy of the bottom of the mth subband at $B = 0$. The expression for Δ_B in Eq. (7) is valid to the terms second-order in the parameter $\hbar\omega_c/(E_2 - E_1)$, where $\omega_c = eB_z/m^*c$. For the field $B_z = 2.5\,\mathrm{T}$ (in the experiment [13] this field corresponds to the filling factor $\nu = 9/2$), one can neglect this correction in the estimation of Δ_B, because $\hbar\omega_c \approx 4\,\mathrm{meV}$, while the gap $E_2 - E_1$ should exceed the Fermi energy E_f measured from the lowest subband; one has $E_f \approx 10\,\mathrm{meV}$ for the 2D electron concentration $N_s = 2.7 \times 10^{11}\,\mathrm{cm}^{-2}$.

IV. ESTIMATES

Based on the experimental data [13], we estimate Δ_{nat} and Δ_B for $B_\| = 0.5\,\mathrm{T}$ (if $\mathbf{B}_\|\|[110]$, this magnetic field converts the resistance from anisotropic to isotropic; at larger $B_\|$ the direction of "easy" conductivity rotates by $90°$). Since the information on the samples is incomplete, we carried out a series of self-consistent calculations by varying the concentration N_a of residual acceptors in GaAs. At $N_a = 10^{14}\,\mathrm{cm}^{-3}$ and N_s taken from [13], the results are

$$\sum_m{}'\frac{|\langle z \rangle_{1m}|^2}{E_m - E_1} \approx 1 \times 10^{-11}\,cm^2/eV, \tag{8}$$

$$\langle \delta(z) \rangle_{11} \approx 1 \times 10^5\,cm^{-1}. \tag{9}$$

For other N_a values (from 10^{13} to $10^{15}\,\mathrm{cm}^{-3}$), the results differ from Eq. (8) and (9) by a factor less than two. Equations (7) and (8) yield the following value for the EM anisotropy induced by magnetic field $B_\| = 0.5\,\mathrm{T}$:

$$\Delta_B = 1.3 \times 10^{-3} = 0.13\%. \tag{10}$$

The parameter α can be determined from the equality $\Delta_{nat} = \Delta_B$ to give

$$\alpha = \frac{0.65 \times 10^{-8}\,cm}{m^*} = 1.1 \times 10^{20}\,cm/g. \tag{11}$$

The two-band approximation with energy gap E_g yields the following estimate for Eq. (3):

$$\alpha \sim \frac{2 \langle c \mid p_x \mid X \rangle D_{0XY} \langle Y \mid p_y \mid c \rangle}{m_0^2 E_g^2} = \frac{D_{0XY}}{m^* E_g}. \tag{12}$$

Thus, it follows from the experimental data [13] and Eqs. (11) and (12) that $D_{0XY} \sim 0.4 \times 10^{-8}\,\mathrm{eV\,cm}$. Let us compare this value with the literature data.

It was found in [20] that different estimates carried out for the GaAs/AlAs heterostructures either on the basis

of pseudopotential and tight-binding calculations or from the comparison with the experiment on the anisotropic exchange splitting of exciton levels in the II-type super-lattices GaAs/AlAs lead to a sizable scatter of the D_{0XY} parameter. The value obtained in [20] lies in the range $(0.35, 0.99) \times 10^{-8}$ eV cm. Using linear interpolation, one obtains the upper bound $D_{0XY} = 0.3 \times 10^{-8}$ eV cm for the GaAs/Al$_{0.3}$Ga$_{0.7}$As heterostructure. This value is in a fair agreement with the value obtained above.

V. DISCUSSION

We can now conclude that the natural anisotropy of EM is likely the mechanism that pins the stripe directions at $B_\parallel = 0$ (see also the end of section 1). It follows from this conclusion that the parameter α entering Eq. (1) is negative, $\alpha < 0$. The competition between the natural anisotropy Δ_{nat} and anisotropy Δ_B induced by the magnetic field $\mathbf{B}_\parallel = (0, B_y)$ makes the 2D electron spectrum at $B_\parallel = 0.5$ T isotropic. As a result, the stripe directions are randomized and the resistance becomes isotropic. On further increase in $B_\parallel$, the magnetic anisotropy prevails and the stripes rotate at $90°$. The role of EM anisotropy in the formation of many-electron anisotropic states can be understood as follows. A 2D electron system with anisotropic EM and isotropic Coulomb interaction is, obviously, equivalent to a 2D electron system with isotropic (cyclotron) mass and anisotropic Coulomb interaction. One can expect that this effective anisotropic interaction is precisely the one which pins the orientation of the striped phase to ensure its observation in the magneto-transport.

For holes, the heterointerface contribution of symmetry C_{2v} (and, hence, responsible for the pinning of the striped phase) is greater than for the conduction band, because it appears in the first-order perturbation treatment [19], whereas the anisotropic EM in Eq. (1) was obtained in the third order. For this reason, one would expect that the hole striped phase is more stable and can form upon filling the lower Landau levels (cf. [22]).

[1] H. Fukuyama, P. M. Platzman, and P. W. Anderson, Phys. Rev. **B19** (1979) 5211.

[2] M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, Phys. Rev. **B54** (1996) 1853.

[3] M. M. Fogler and A. A. Koulakov, Phys. Rev. **B55** (1997) 9326.

[4] R. Moessner and J. T. Chalker, Phys. Rev. **B54** (1996) 5006.

[5] G. R. Aizin and V. A. Volkov, Zh. Eksp. Teor. Fiz. **87** (1984) 1469 [Sov. Phys. JETP **60** (1984) 844]; Zh. Eksp. Teor. Fiz. **92** (1987) 329 [Sov. Phys. JETP **65** (1987) 188].

[6] G. R. Aizin and V. A. Volkov, Fiz. Tekh. Poluprovodn. (Leningrad) **19** (1985) 1780 [Sov. Phys. Semicond. **19** (1985) 1094].

[7] G. R. Aizin and V. A. Volkov, Fiz. Tverd. Tela (Leningrad) **27** (1985) 475 [Sov. Phys. Solid State **27** (1985) 290].

[8] A. H. MacDonald and M. P. A. Fisher, Phys. Rev. **B61** (2000) 5724.

[9] F. von Oppen, B. I. Halperin, and A. Stern, Phys. Rev. Lett. **84** (2000) 2937.

[10] M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **82** (1999) 394.

[11] R. R. Du, D. C. Tsui, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Solid State Commun. **109** (1999) 389.

[12] W. Pan, R. R. Du, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **83** (1999) 820.

[13] M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **83** (1999) 824.

[14] J. P. Eisenstein, M. P. Lilly, K. B. Cooper, L. N. Pfeiffer, and K. W. West, cond-mat/0003405.

[15] T. Jungwirth, A. H. MacDonald, L. Smrčka, and S. M. Girvin, Phys. Rev. **B60** (1999) 15574.

[16] T. D. Stanescu, I. Martin, and P. Phillips, Phys. Rev. Lett. **84** (2000) 1288.

[17] H. Kroemer, cond-mat/9901016.

[18] T. Ando, A. Fowler, and F. Stern, Rev. Mod. Phys. **54** (1982) 437.

[19] E. E. Takhtamirov and V. A. Volkov, Zh. Eksp. Teor. Fiz. **116** (1999) 1843 [JETP **89** (1999) 1000].

[20] E. L. Ivchenko, A. Yu. Kaminski, and U. Rössler, Phys. Rev. **B54** (1996) 5852.

[21] O. Krebs, W. Seidel, J. P. André, D. Bertho, C. Jouanin, and P. Voisin, Semicond. Sci. Technol. **12** (1997) 938.

[22] M. Shayegan, H. C. Manoharan, S. J. Papadakis, and E. P. De Poortere, Physica E, **6** (2000) 40.